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Analogue correction method of errors and its application to numerical weather prediction*

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In this paper, an analogue correction method of errors (ACE) based on a complicated atmospheric model is further developed and applied to numerical weather prediction (NWP). The analysis shows that the ACE can effectively reduce model errors by combining the statistical analogue method with the dynamical model together in order that the information of plenty of historical data is utilized in the current complicated NWP model. Furthermore, in the ACE, the differences of the similarities between different historical analogues and the current initial state are considered as the weights for estimating model errors. The results of daily, decad and monthly prediction experiments on a complicated T63 atmospheric model show that the performance of the ACE by correcting model errors based on the estimation of the errors of 4 historical analogue predictions is not only better than that of the scheme of only introducing the correction of the errors of every single analogue prediction, but is also better than that of the T63 model.

Keywords: numerical weather prediction, analogue correction method of errors, reference state, analogue-dynamical model

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1. Introduction

It is well known that the deterministic limit of atmospheric predictability is generally shorter than 2–3 weeks.[1] In fact, both the performances of models and the qualities of observed data have been greatly improved in the past 30 years of numerical weather prediction (NWP). However, the valid forecast period of atmospheric circulation averages no more than 10 days and the improvements in prediction skill levels appear more within 10 days, but little after 10 days.[2] Consequently, new approaches need urgently to be explored in order to further improve the skill of NWP, especially for daily weather forecast beyond 10 days.

Introducing past historical data to numerical prediction is a significant and feasible approach proposed early in the 1950s.[3] On one hand, based on different principles and criteria, a series of innovative prediction methods with multi-time levels have already been put forward by using atmospheric evolvement data in the near past, such as, the multi-time model,[4] the self-memorial forecast model,[5–7] the method of forming optimal initial members of ensemble forecast,[8] and the retrospective scheme with multi-time levels,[9–11] etc. These methods have been widely applied to the numerical prediction and showed good capabilities in improving prediction skill.

On the other hand, in order to further utilize the information of abundant historical climate data, there have been some statistical studies on the short-term climate prediction and atmospheric predictability by using the atmospheric analogy principle.[12–16] To effectively combine numerical prediction model with the subjective experiences of forecasters in analogue prediction, the dynamical prediction field may be as-

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sumed as a small disturbance of the historical analogue field so that synoptic experiences are introduced to numerical prediction.[17] In terms of this basic principle, some analogue-dynamical models (ADMs) have been established based on the analogue-deviation versions of the simple quasi-geostrophic models by considering the analogy evolvement of atmospheric circulation anomalies. Due to the compensating effect to model errors from historical analogues, these ADMs have higher accuracy than ordinary dynamical models, which has been documented by monthly and seasonal prediction experiments.[18−20]

But it is quite difficult to directly rebuild the analogue-deviation version of a complicated numerical model. Actually, a theoretically equivalent approach based on error correction has been introduced to establish the equivalent ADM[21,22] for the T63 monthly dynamical extended-range forecast (DERF) model of the National Climate Center (NCC) in China Meteorological Administration (CMA). The equivalent T63 ADM also has higher accuracy than the original model and only need diagnose error terms by running the prediction model with single historical analogue. However, there usually exist many suitable historical analogues and it may be more reasonable to combine all their information. Furthermore, the differences of the similarities between different historical analogues and the current initial state should also be considered, where the better similarity is, the larger contribution to current forecast will be.

In a previous study,[22] an analogue correction method of errors (ACE) has been theoretically put forward by considering the estimation of model errors in terms of the differences of the similarities between different historical analogues and initial state. So we will further develop and apply the ACE on a complicated operational prediction model. Next, in view of the fact that many studies have been focused on monthly or seasonal prediction previously, here we will primarily examine the application of the ACE to prediction on weather timescale with some experiments.

2. Basic principle and method

In general, numerical prediction model can be mathematically expressed as follows:

$$\frac{\partial \Psi}{\partial t} + L(\Psi) = 0, \tag{1}$$

$$\Psi(r,0) = G(r), \tag{2}$$

where $\Psi(r,t)$ is the model state vector to be predicted, r is the vector in the spatial coordinates, t is time, and L is the differential operator of Ψ, which is corresponding to real numerical model and usually nonlinear. Similarly, the exact model that real atmosphere satisfies can be written as

$$\frac{\partial \Psi}{\partial t} + L(\Psi) = E(\Psi), \tag{3}$$

in which E is the error term and stands for the process that actually exists but is not described or exactly described in (1), and just reflects the errors of real numerical model. Then historical data may be regarded as a series of special solutions or their functions of (3) and (2).

According to the basic principle of the analogue-dynamical method,[17] Ψ can be divided into the analogue reference state (or reference state for short, abbreviated as RS) $\tilde{\Psi}$ and the analogue disturbance state (or disturbance state for short, abbreviated as DS) Ψ', namely $\Psi = \tilde{\Psi} + \Psi'$, where $\tilde{\Psi}$ is selected from historical data in terms of the similarities between the RSs and the current initial state $G(r)$. The RS satisfies the following equations:

$$\frac{\partial \tilde{\Psi}}{\partial t} + L(\tilde{\Psi}) = E(\tilde{\Psi}), \tag{4}$$

$$\tilde{\Psi}(r,0) = \tilde{G}(r). \tag{5}$$

By subtracting (4) and (5) from (3) and (2) respectively, the exact equation that the DS satisfies is obtained:

$$\frac{\partial \Psi'}{\partial t} + L(\tilde{\Psi} + \Psi') - L(\tilde{\Psi}) = E(\tilde{\Psi} + \Psi') - E(\tilde{\Psi}), \tag{6}$$

$$\Psi'(r,0) = G(r) - \tilde{G}(r). \tag{7}$$

Similarly, substituting $\Psi = \tilde{\Psi} + \Psi'$ and $\tilde{\Psi}$ into (1), and subtracting the latter from the former, we obtain the analogue-deviation equation (ADE):

$$\frac{\partial \Psi'}{\partial t} + L(\tilde{\Psi} + \Psi') - L(\tilde{\Psi}) = 0. \tag{8}$$

The model based on Eq.(8) is just called the ADM, viz. the analogue-deviation version of original model. Furthermore, we can Taylor expand $E(\tilde{\Psi} + \Psi')$ to first order around $\tilde{\Psi}$ as follows:[22]

$$E(\Psi) = E(\tilde{\Psi} + \Psi') \equiv E(\tilde{\Psi}) + \Psi' D|_{\tilde{\Psi}},$$

where D represents the sum of the partial differentials of E with respect to every component of Ψ. As we can see, when $D|_{\tilde{\Psi}}$ is bounded and $\|\Psi'\|$ is small enough, it is not difficult to obtain

$$\|E(\tilde{\Psi} + \Psi') - E(\tilde{\Psi})\| << \|E(\Psi)\|.$$

This suggests that the ADM on the basis of (8) has fewer model errors and higher accuracy than the ordinary numerical model on the basis of (1), although the two kinds of models are both inexact. By selecting the RS $\tilde{\Psi}$ of current initial state firstly, the DS Ψ' can be calculated in terms of (8) and (7), and the current forecast Ψ can be obtained by $\Psi = \tilde{\Psi} + \Psi'$.

In fact, for complicated operational models, it is quite difficult to directly establish a new analogue-deviation model in terms of (8). In view of the fact that the RS $\tilde{\Psi}$ in historical data is known, $E(\tilde{\Psi})$ may be diagnosed by the left-side terms of (4) under the condition that observed errors are far smaller than model errors. Thus, provided that the error term $E(\Psi)$ on the right side of (3) is directly estimated with the error term $E(\tilde{\Psi})$ on the right-hand side of (4), we can obtain:

$$\frac{\partial \Psi}{\partial t} + L(\Psi) = \frac{\partial \tilde{\Psi}}{\partial t} + L(\tilde{\Psi}). \qquad (9)$$

Evidently, Eq.(9) is mathematically equivalent to (8) and is named as the analogue-correction equation because Eq.(9) looks like appending an analogue correction term of errors to (1). So we may add a correction process into the original model to reduce model errors.

Eq.(8) states for a set of the deviation equations and implies that the rebuilding of an ADM is quite difficult for complicated models, whereas Eq.(9) needs not change the original model but only diagnoses the two terms on the right side of (9) by running the prediction model with historical analogue data. Thus, it can be seen that the analogue-dynamical method on the basis of (9) is practically superior to the analogue deviation model based on (8). An equivalent ADM, in which the error terms of current model prediction are estimated by the error terms of single historical analogue prediction according to the right-side terms of (9), has already been preliminarily established based on the T63 DERF model of NCC/CMA.[21] Such a T63 ADM is actually made up of the original dynamical prediction model and the diagnostic model which is introduced to the estimation of the analogue correction terms of model errors.

However, there usually exist many suitable historical analogues and it may not be very reasonable to use single RS. Furthermore, the differences of the similarities between different historical analogues and the initial state of current prediction are not considered in the preliminary T63 ADM. The better similarity is, the larger contribution to current forecast will be. Thus, in terms of such idea, an analogue correction method of errors has been theoretically put forward by considering the differences of different analogues.[22] Based on this previous study, here we further develop and apply the ACE on a complicated model.

The following formula[22] is introduced to diagnose the correction term of errors:

$$E(\Psi_0) = \sum_{j=1}^{m} a_j E(\tilde{\Psi}_j), \qquad (10)$$

where Ψ_0 is the current initial state, $\tilde{\Psi}_j$ represents the No. j selected historical analogue or RS, m is the number of RSs, and a_j stands for the No. j standardized weighted coefficient and is defined as

$$a_j = b_j \Big/ \sum_{j=1}^{m} b_j,$$

where b_j is the undefined coefficient associated with the degrees of the similarities between different historical analogues and current initial state. In this paper, such similarities between atmospheric states will be measured by utilizing a simple Euclid distance function expressed as

$$b = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i - Y_i)}, \qquad (11)$$

which is called as analogy criterion or analogue index. X and Y are the same variable with n degrees of freedom at any two times. Here, the global 500-hPa geopotential height is used to calculate the analogy criterion. The more similar two fields are, the smaller b between them is.

In brief, so long as the similarities between historical analogues and current initial state are to some extent good, theoretically, the ACE can effectively reduce prediction errors of model by introducing the error correction based on the information of multi historical analogues.

3. Prediction experiments

In the following sections, we will apply the ACE to the experiments of NWP. The prediction with the T63 dynamical model (referred to as T63 model hereafter) is regarded as control experiment, and the prediction with the ACE is regarded as the contrast one in which we select m=4 in Eq.(10). Specially, it may be regarded as the simplest ACE (hereafter as sACE 1–4) that the four single historical analogues or RSs are used to estimate the error terms (viz. m=1) respectively, which is similar to the scheme of Bao *et al.*[21] A

35-year historical dataset in the period of 1968–2002 is taken from the daily and 6-hourly NCEP/NCAR Reanalysis data, and persisted sea surface temperature anomalies are used in the model integrations. Twelve cases in 2002 are selected for prediction experiments, and the integrating period of every case month is 30 days which starts from 12:00 UTC of the end day of the last month. Based on the analogy criterion of (11), we select the first four most similar RSs for every case from the 35 historical analogues which are identified in the same season of every year respectively. The anomaly correlation coefficient (ACC) and the root mean square error (RMSE) are utilized for the verifications of the experiment results.

3.1. Prediction of daily circulation

Firstly, we verify the daily prediction skill of three schemes, namely the T63 model, ACE, and sACE 1–4, in the global areas (90°N–90°S), the northern hemisphere extratropics (NHE; 20°N–90°N), the tropics (20°N–20°S), and the southern hemisphere extratropics (SHE; 20°S–90°S). Figure 1 gives the daily ACCs averaged over 12 cases between the prediction and the observation of 500hPa height in the different areas with respect to the three schemes respectively.

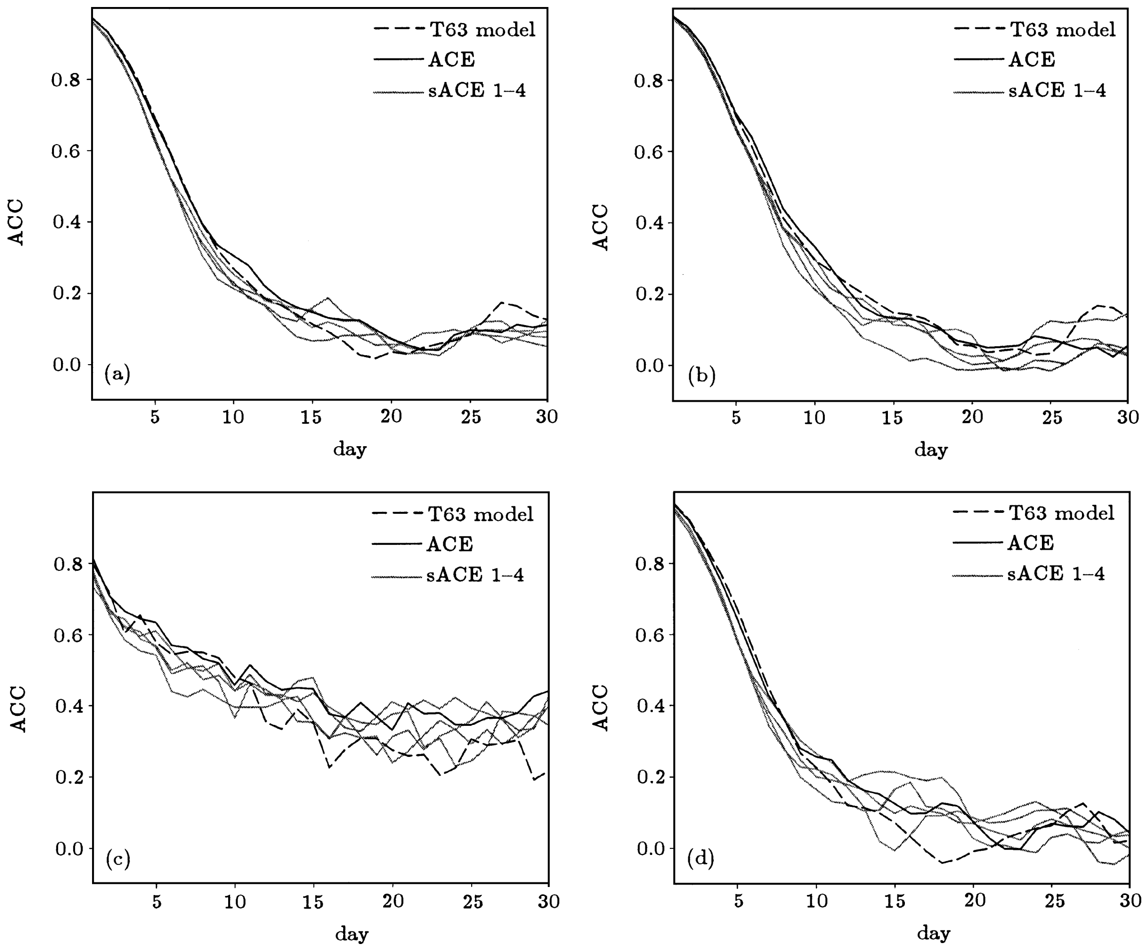

Fig.1. The daily ACCs averaged over 12 cases between the prediction and the observation of 500 hPa height in the global areas (a), the NHE (b), the tropics (c), and the SHE (d), respectively.

It can be seen from Fig.1 that the ACE displays higher ACCs than the T63 model during the 30-day period except for the first 7 days in the SHE and the last 5 days in the NHE. On the other hand, most of the forecasts of the sACE 1–4 have improved daily skill from about 10-day to 25-day leading time except for the NHE compared with those of the T63 model, but all of the forecasts of the sACE 1–4 have almost lower ACCs than those of the T63 model during the first 10-day leading time (see also the 1st decad ACCs in Tables 1–4). Also, the ACE curves span on top of the sACE 1–4 curves in most of the 30-day leading time, which indicates that the prediction skill of the ACE is higher than that of the sACE 1–4.

Figure 2 presents the daily RMSEs averaged over 12 cases. The situations of the RMSE curves are almost opposite to those of the ACC curves. That is to say, the ACE displays smaller RMSEs than the T63 model during the 30-day period except the first 7 days in the SHE and the last 5 days in the NHE. Furthermore, during the first 10 days, almost all of the forecasts of the sACE 1–4 have bigger RMSEs than those of the T63 model (see also the 1st decad RMSEs in Tables 1–4), whereas most of the forecasts of the sACE 1–4 have reduced daily forecast errors from about 10-day to 25-day leading time except the NHE compared with those of the T63 model. Also, the RMSE curves span on bottom of the sACE 1–4 curves in most of 30-day leading time, which indicates that the forecast errors of the ACE are smaller than those of the sACE 1–4.

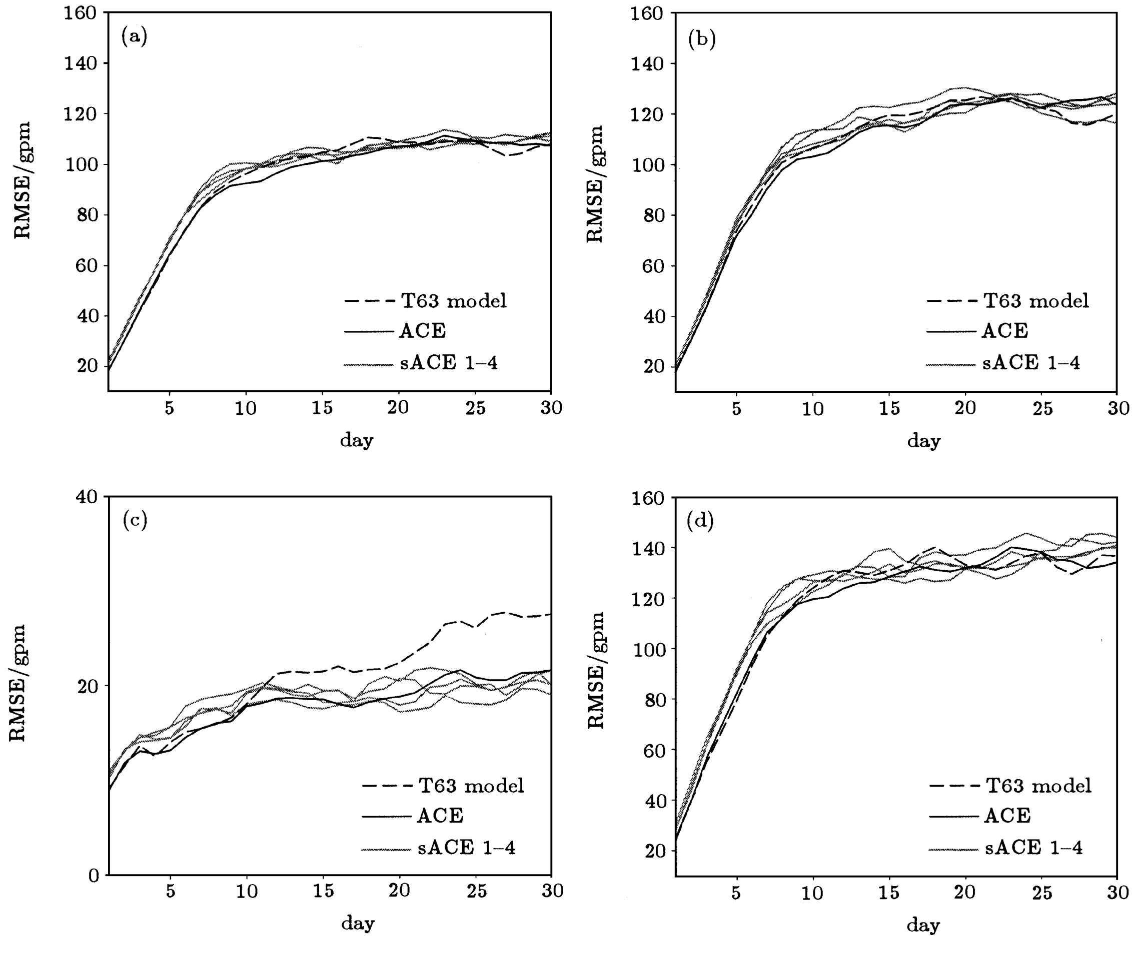

Fig.2. The same as Fig.1, but for the daily RMSEs.

3.2. Prediction of decade-mean circulation

The above prediction results of daily circulation show that the performance of the ACE by correcting error terms based on the estimation of the errors of four historical analogue predictions is not only better than that of the sACE 1–4 by only introducing the correction of the errors of every single analogue prediction, but is also better than that of the T63 model. However, such improvements of the performance of the ACE show clear distinctions in different leading times. For example, the improvements of the prediction skill of the ACE in the global areas are characterized by the increases of ACCs during a 8- to 21-day span of leading time. Thus, we will further examine the prediction of decad-mean circulation of the three schemes.

Table 1. Decad and monthly ACCs and RMSEs of 500 hPa height in the global areas, where the T63 model and ACE represent the average of 12 cases, and the sACE 1–4 stands for the average of 48 cases.

Forecast scheme	ACC				RMSE (gpm)			
	1st decad	2nd decad	3rd decad	Monthly	1st decad	2nd decad	3rd decad	Monthly
T63 model	0.77	0.21	0.19	0.43	39.29	68.28	72.58	43.17
ACE	0.78	0.24	0.17	0.46	37.85	65.11	73.17	41.43
sACE 1–4	0.75	0.23	0.14	0.43	40.87	66.80	73.72	42.68

Table 2. The same as Table 1, but for the northern hemisphere extratropics.

Forecast scheme	ACC				RMSE (gpm)			
	1st decad	2nd decad	3rd decad	Monthly	1st decad	2nd decad	3rd decad	Monthly
T63 model	0.79	0.20	0.14	0.41	45.78	84.46	87.73	51.44
ACE	0.81	0.22	0.10	0.48	43.46	79.98	89.60	50.19
sACE 1–4	0.77	0.18	0.06	0.40	45.11	81.83	85.99	50.45

Table 3. The same as Table 1, but for the tropics.

Forecast scheme	ACC				RMSE (gpm)			
	1st decad	2nd decad	3rd decad	Monthly	1st decad	2nd decad	3rd decad	Monthly
T63 model	0.71	0.51	0.40	0.63	10.18	17.08	23.13	14.79
ACE	0.73	0.64	0.58	0.76	9.43	12.86	15.88	10.17
sACE 1–4	0.68	0.60	0.56	0.72	10.37	12.77	13.81	9.59

Table 4. The same as Table 1, but for the southern hemisphere extratropics.

Forecast scheme	ACC				RMSE (gpm)			
	1st decad	2nd decad	3rd decad	Monthly	1st decad	2nd decad	3rd decad	Monthly
T63 model	0.73	0.21	0.12	0.41	47.78	78.18	84.42	50.34
ACE	0.74	0.22	0.13	0.43	46.56	77.08	85.38	49.13
sACE 1–4	0.70	0.25	0.13	0.47	51.87	78.99	90.59	51.52

Tables 1–4 list the decad-mean ACCs and RMSEs averaged over 12 cases of 500 hPa height in the different areas, respectively. We can see that in the four areas, almost all of the three-decade ACCs of the ACE are higher than those of the T63 model, and the corresponding RMSEs are almost always smaller than those of the T63 model, while the ACCs in the NHE and the RMSEs in the NHE and SHE are exceptions in the 3rd decad. Such exceptions cause the slightly lower ACC and bigger RMSE of the 3rd decad in the global areas compared with those of the T63 model.

Similar to Figs. 1 and 2, all of the forecasts of the sACE 1–4 in the 1st decad of Tables 1–4 have lower ACCs and almost always bigger RMSEs than those of the T63 model, but in the 2nd and 3rd decads, the prediction skill of the former is higher than that of the latter, especially in the tropics and SHE. On the other hand, as we can see from Figs.1 and 2, Tables 1–4 also show that the prediction results of the ACE are corresponding to higher ACCs and smaller RMSEs than those of the sACE 1–4 except for some exceptions, such as the RMSEs in the 3rd decad in the NHE, the ACCs in the 2nd decad in the SHE, etc.

3.3. Prediction of monthly mean circulation

In order to comprehensively verify the performance of the ACE, we also examine the prediction of monthly mean circulation of the three schemes. It can be seen from the monthly verifications in Tables 1–4 that the ACE has higher ACCs and smaller RMSEs averaged over 12 cases than the T63 model, especially in the NHE and tropics. Moreover, the performances of the sACE 1–4 are better than those of the T63 model on the monthly timescale except that they have slightly lower ACC in the NHE and bigger RMSE in the SHE than the T63 model. Similar to the daily and decad-mean prediction, the ACE exhibits stronger capabilities in the prediction of monthly mean circulation than the sACE 1–4 except that the sACE 1–4 have slightly higher ACC in the SHE and smaller

RMSE in the tropics than the ACE. Also, it is worthy to note that the performances of the three schemes are different in each area and the prediction scores over the global area may be regarded as general verifications. Figures 3 and 4 further give the twelve monthly ACCs and RMSEs in 2002 respectively.

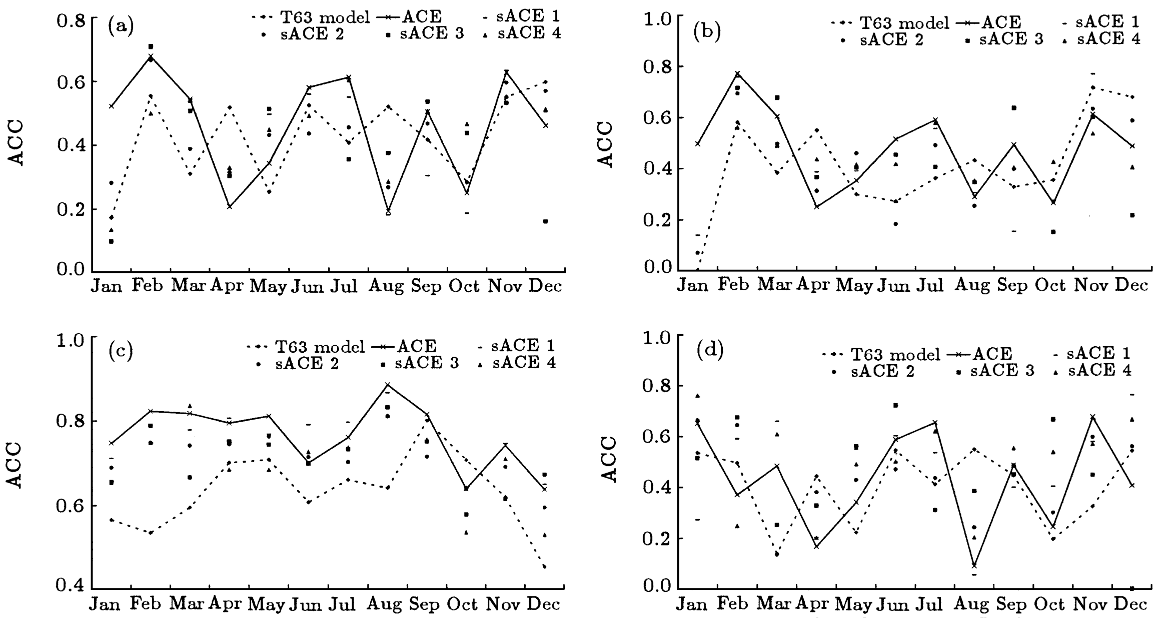

Fig.3. ACCs between the prediction and the observation of the monthly mean 500 hPa height in 2002 in the global areas (a), the NHE (b), the tropics (c), and the SHE (d), respectively.

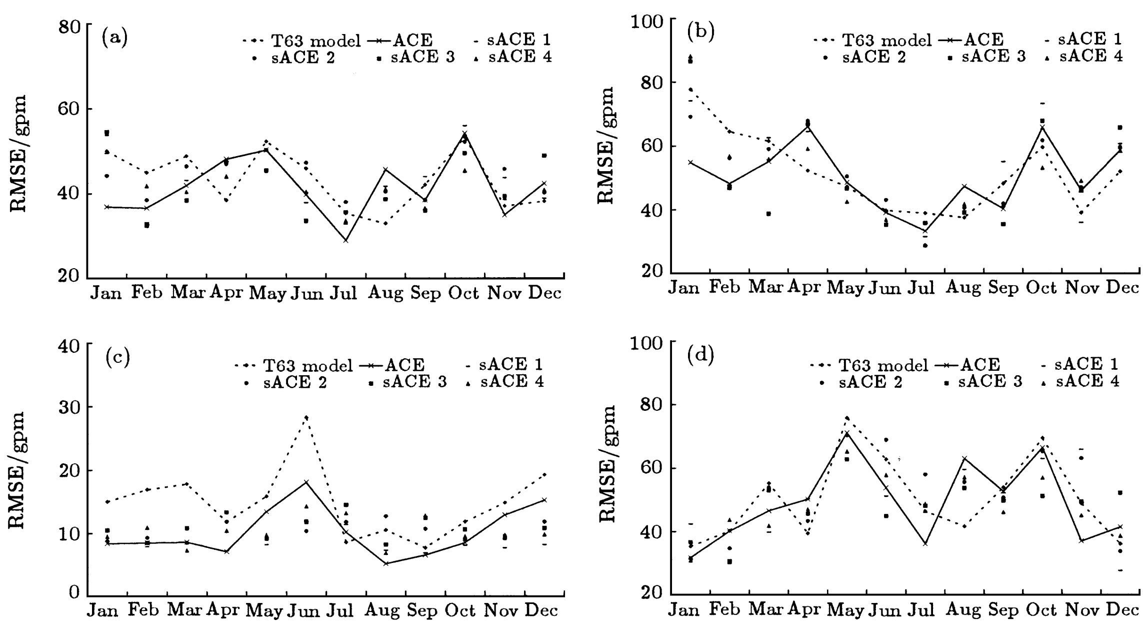

Fig.4. The same as Fig.3, but for the RMSEs.

We can see that the three ACE curves in Fig.3(a), (b), and (d) have quite similar seasonal variations, and that the same similarity also appears between the three ACE curves in Fig.4(a), (b), and (d). Furthermore, the number of cases in which the ACC of the ACE is higher than that of the T63 model is 8, 7, 11, and 8 in Fig.3(a)–(d), respectively, and the number of cases in which the RMSE of the ACE is smaller than that of the T63 model is 8, 6, 11, and 9 in Fig.4(a)–(d), respectively. In the same way, relative to the T63 model, the two sets of numbers of the sACE 1–4 may also be obtained, namely 26, 24, 38, and 31 for ACC, and 25, 24, 39, and 28 for RMSE. All these results indicate that compared with the T63 model, the improvement of the prediction capabilities of the ACE and sACE 1–4 are decreasing in the order of the tropics, SHE, and NHE. Evidently, in most of the cases, the ACE and sACE 1–4 have improved ACCs and reduced RMSEs compared with the T63 model.

4. Conclusion

An analogue correction method of errors (ACE) based on a complicated atmospheric model has been further developed and applied to NWP. The ACE can effectively reduce model errors without the need of building new models, in which the statistical analogue method is closely combined with dynamical model together, and the information of plenty of historical data can be adequately utilized to improve the dynamical prediction of the current complicated NWP model. Furthermore, the differences of the similarities between different historical analogues and current initial state have been considered as the weights for estimating model error terms in such an ACE.

The results of the 12-case prediction experiments on a complicated T63 DERF model of NCC/CMA have shown that the performance of the ACE by correcting model errors based on the estimation of the errors of four historical analogue predictions is not only better than that of the sACE 1–4 by only introducing the correction of the errors of every single analogue prediction, but is also better than that of the T63 model. In the majority of cases, on whether the daily, decad, or monthly timescale, the ACE has considerable capabilities of improving prediction skill and reducing forecast errors compared with either the T63 model or sACE 1–4 in different areas. Also, in most cases the sACE 1–4 can improve prediction level in comparison with the T63 model. Moreover, the performances of the three schemes display obvious distinctions in different areas and the superiority of the ACE is primarily characterized by the improvement of daily forecast skill and the reduction of prediction errors after about one week leading time.

Evidently, further theoretical studies and more prediction experiments are necessary for the improvement of the performance of the ACE. More effective analogy criteria should be introduced to selecting analogue reference states. The number of historical analogues need also be reasonably determined for the optimal prediction. These problems will be studied in further work.

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